

Hard Multi-Jet Predictions using High Energy Factorisation

Jeppe R. Andersen (PH-TH)

in collaboration with
Vittorio Del Duca (INFN, Frascati) & Chris White (NIKHEF)

CERN
January 25, 2008

What, Why, How?

What?

Develop a framework for reliably calculating many-parton rates inclusively (ensemble of 2, 3, 4, ... parton rates) and in a flexible way (jets, W+jets, Higgs+jets, ...)

What, Why, How?

What?

Develop a framework for reliably calculating many-parton rates inclusively (ensemble of 2, 3, 4, ... parton rates) and in a flexible way (jets, W+jets, Higgs+jets, ...)

Why?

$(n + 1)$ -jet rate not necessarily small compared to n -jet rate
Partons not in observed jets can masquerade as \cancel{p}_\perp

What, Why, How?

What?

Develop a framework for reliably calculating many-parton rates inclusively (ensemble of 2, 3, 4, ... parton rates) and in a flexible way (jets, W+jets, Higgs+jets, ...)

Why?

$(n + 1)$ -jet rate not necessarily small compared to n -jet rate
Partons not in observed jets can masquerade as \cancel{p}_\perp

How?

Factorisation of QCD Amplitudes in the High Energy Limit.
New Technique. Validation.

Why Study Multi-jet Observables?

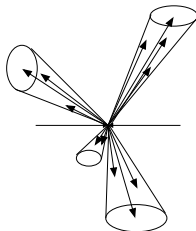
What is a jet (-algorithm)?

Organisational principle for events, which allows for a relation between the perturbative calculations with a few, hard partons (**theory**) and the many-hadron events observed in **experiments**.

Why Study Multi-jet Observables?

What is a jet?

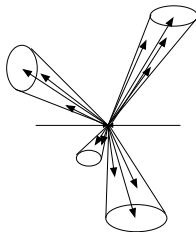
- Experimentally: Collimated spray of (colour s.) particles



Why Study Multi-jet Observables?

What is a jet?

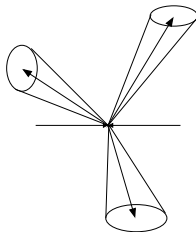
- Experimentally: Collimated spray of (colour s.) particles
- Theoretically:
 - ① LO: A single coloured particle (parton \leftrightarrow hadron duality)
 - ② NLO: Possibly two particles
 - ③ Parton Shower and Hadronisation MC (a la Herwig): Collimated spray of (colour singlet) particles



Why Study Multi-jet Observables?

What is a jet?

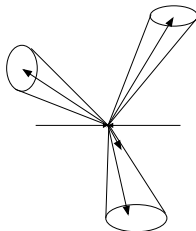
- Experimentally: Collimated spray of (colour s.) particles
- Theoretically:
 - 1 LO: A single coloured particle (parton \leftrightarrow hadron duality)
 - 2 NLO: Possibly two particles
 - 3 Parton Shower and Hadronisation MC (a la Herwig):
Collimated spray of (colour singlet) particles



Why Study Multi-jet Observables?

What is a jet?

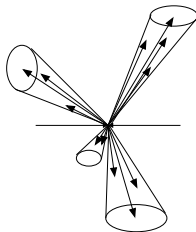
- Experimentally: Collimated spray of (colour s.) particles
- Theoretically:
 - 1 LO: A single coloured particle (parton \leftrightarrow hadron duality)
 - 2 **NLO: Possibly two particles**
 - 3 Parton Shower and Hadronisation MC (a la Herwig):
Collimated spray of (colour singlet) particles



Why Study Multi-jet Observables?

What is a jet?

- Experimentally: Collimated spray of (colour s.) particles
- Theoretically:
 - LO: A single coloured particle (parton \leftrightarrow hadron duality)
 - NLO: Possibly two particles
 - Parton Shower and Hadronisation MC (a la Herwig):
Collimated spray of (colour singlet) particles
But tends to describe only few hard jets



Why Study Multi-jet Observables?

What is a jet?

- Experimentally: Collimated spray of (colour s.) particles
- Theoretically:
 - ① LO: A single coloured particle (parton \leftrightarrow hadron duality)
 - ② NLO: Possibly two particles
 - ③ Parton Shower and Hadronisation MC (a la Herwig):
Collimated spray of (colour singlet) particles
But tends to describe only few hard jets

The current discussion is independent on the exact jet-definition (kt , $S/\text{Scone}, \dots$), although some reasonable (i.e. IR-safe) algorithm obviously is necessary to guarantee the relation between theoretical calculation and experimental observation

Why Study Multi-jet Observables?

We don't have a choice!

- 1 Many BSM (e.g. SUSY) particles will have *decay chains* involving the production of jets (e.g. 4 jets + p_T). Calculation of signal is easy (one process), SM contribution is very hard (several processes).
- 2 **All** LHC processes involves QCD-charged particles; sometimes the (n+1)-jet cross section is as large as the n-jet cross section!
- 3 It is a challenge we cannot ignore !

Why Study Multi-jet Observables?

Just a few important examples

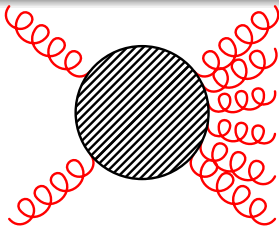
- 1 **Pure Multi-jets**
- 2 $W + (n \geq 2)$ jets
- 3 Higgs + 2 jets

Will discuss how all these observables can be described in a framework tailored to the description of multiple, also (but not limited to) hard gluon emission

Why Study Multi-jet Observables?

Pure Multi-jets : High Rate

- 1 High rate: Possibility to look for interesting QCD effects in **new corners of phase space** and to further our understanding of the behaviour of field theories.
(Not just looking for 2 high p_{\perp} jets in search of quark compositeness, but now have energy for several hard jets)
- 2 Partons escaping detection as jets (below p_{\perp} -threshold) can mimic missing energy



Why Study Multi-jet Observables?

Just a few important examples

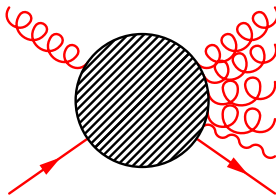
- 1 Pure Multi-jets
- 2 $W + (n \geq 2)$ jets
- 3 Higgs + 2 jets

Will discuss how all these observables can be described in a framework tailored to the description of multiple, also (but not limited to) hard gluon emission

Why Study Multi-jet Observables?

$W + (n \geq 2)$ jets

- 1 Important for various new physics signatures involving leptons, jets, and missing transverse energy
- 2 Enters on the “wish-list” for higher order calculations in preparation for LHC physics
- 3 Dominated by diagrams with an incoming quark at lowest order \rightarrow multi-jet rates have larger relative contribution



Why Study Multi-jet Observables?

Just a few important examples

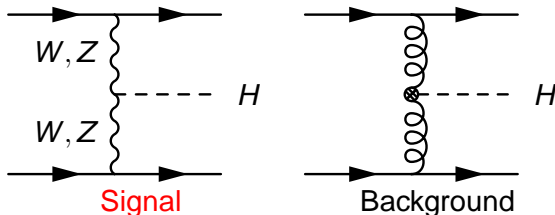
- 1 Pure Multi-jets
- 2 $W + (n \geq 2)$ jets
- 3 **Higgs + 2 jets**

Will discuss how all these observables can be described in a framework tailored to the description of multiple, also hard gluon emission

Why Study Multi-jet Observables?

$H + (n \geq 2)$ jets

- 1 When(!) a fundamental scalar has been found at the LHC we need to determine whether this one is responsible for the observed EWSB
- 2 Determine the couplings to Z or W by studying the angular distribution of the jets



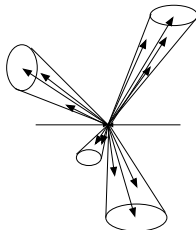
Important to understand the behaviour of the QCD process in order to separate the two channels

Why Study Multi-jet Observables?

Just a few important examples

- 1 Pure Multi-jets
- 2 $W + (n \geq 2)$ jets
- 3 Higgs + 2 jets

Will discuss how all these observables can be described in a framework tailored to the description of multiple, also hard gluon emission



Do we need a new approach?

Already know how to calculate...

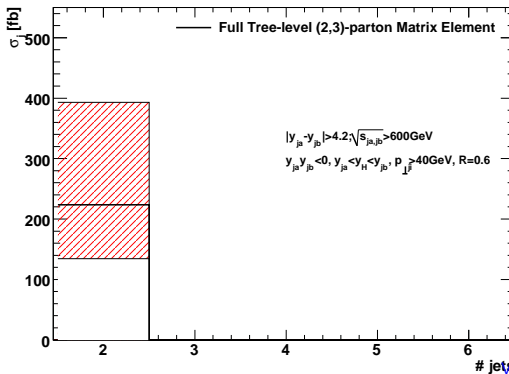
- Shower MC: at most $2 \rightarrow 2$ "hard" processes with additional parton shower
- Flexible Tree level calculators:
MadGraph, AlpGen, SHERPA, ...
Allow most $2 \rightarrow 4$, some $2 \rightarrow 5$ processes (and 6 constrained) to be calculated at tree level.
Interfaced with Shower MC makes for a powerful mix!
- MCFM: Many relevant $2 \rightarrow 3$ processes at up to NLO (i.e. including $2 \rightarrow 4$ -contribution).
- ... ⟨your favourite method here⟩

Could all be labelled "Standard Model contribution", but give vastly different results depending on the question asked!

All Order Resummation Necessary?

Are tree-level (or generally fixed order) calculation always sufficient?

Sometimes the $(n + 1)$ -jet rate is as large as the n -jet rate
Higgs Boson plus n jets at the LHC at leading order



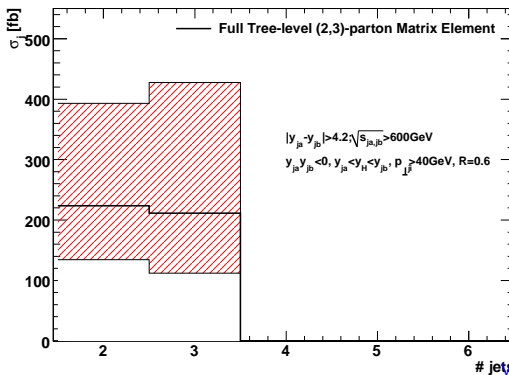
Del Duca, C. White, JRA

Indication that we need to go further! However, fixed order tools exhausted ($2 \rightarrow 3$ with a massive leg at two loops untenable!).

All Order Resummation Necessary?

Are tree-level (or generally fixed order) calculation always sufficient?

Sometimes the $(n + 1)$ -jet rate is as large as the n -jet rate
Higgs Boson plus n jets at the LHC at leading order



Del Duca, C. White, JRA

Indication that we need to go further! However, fixed order tools exhausted ($2 \rightarrow 3$ with a massive leg at two loops untenable!).

Resummation

Consider the **perturbative expansion** of an observable

$$R = r_0 + r_1 \alpha_s + r_2 \alpha_s^2 + r_3 \alpha_s^3 + r_4 \alpha_s^4 + \dots$$

Fixed order pert. QCD will calculate a fixed number of terms in this expansion. r_n may contain **large logarithms** so that $\alpha_s \ln(\dots)$ is large.

$$\begin{aligned} R &= r_0 + \left(r_1^{LL} \ln(\dots) + r_1^{NLL} \right) \alpha_s + \left(r_2^{LL} \ln^2(\dots) + r_2^{NLL} \ln(\dots) + r_2^{SL} \right) \alpha_s^2 + \dots \\ &= r_0 + \sum_n r_n^{LL} (\alpha_s \ln(\dots))^n + \sum_n r_n^{NLL} \alpha_s (\alpha_s \ln(\dots))^n + \text{sub-leading terms} \end{aligned}$$

Replace the perturbative parameter α_s with $\alpha_s \ln(\dots)$. Useful if **the terms** can be **summed to all orders** in the pert. expansion (LLA).

Factorisation of QCD Matrix Elements

Factorisation of QCD Matrix Elements

It is **well known** that QCD matrix elements **factorise** in certain kinematical limits:

Soft limit \rightarrow **eikonal approximation** \rightarrow enters all parton shower (and much else) resummation.

Like all good limits, the eikonal approximation is applied **outside its strict region of validity**.

Factorisation of QCD Matrix Elements

It is **well known** that QCD matrix elements **factorise** in certain kinematical limits:

Soft limit \rightarrow **eikonal approximation** \rightarrow enters all parton shower (and much else) resummation.

Like all good limits, the eikonal approximation is applied **outside its strict region of validity**.

To boldly go...

Previously in another CERN seminar series:

A wise man said...

“Use known results to gain deeper insights...”

young* postdoc

“Use insight to gain yet unknown results...”

New approach using a less well-known factorisation of amplitudes in another kinematical limit.

Will discuss validation**

*or so I prefer to think

**to validate: show how well it works!

To boldly go...

Previously in another CERN seminar series:

A wise man said...

“Use known results to gain deeper insights...”

young* postdoc

“Use insight to gain yet unknown results...”

New approach using a less well-known factorisation of amplitudes in another kinematical limit.

Will discuss validation**

*or so I prefer to think

***to validate: show how well it works!

To boldly go...

Previously in another CERN seminar series:

A wise man said...

“Use known results to gain deeper insights...”

young* postdoc

“Use insight to gain yet unknown results...”

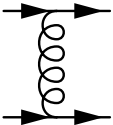
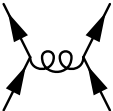
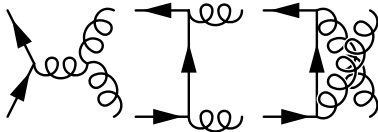
New approach using a less well-known factorisation of amplitudes in another kinematical limit.

Will discuss validation**

*or so I prefer to think

**to validate: show how well it works!

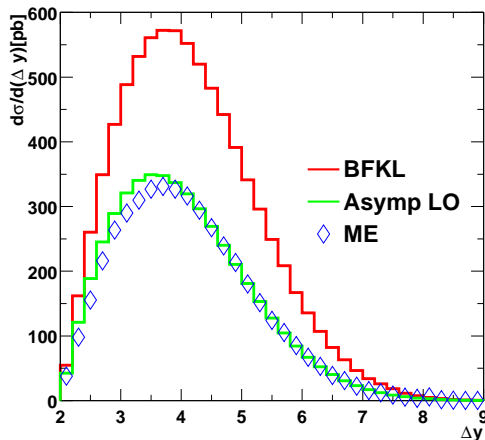
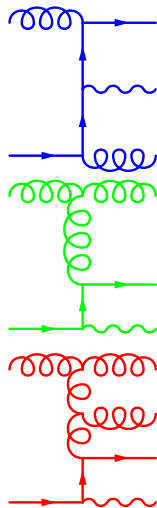
High Energy Factorisation - t -channel dominance

Process	Diagrams	$\overline{\sum} \mathcal{M} ^2 / g^4$
$qq' \rightarrow qq'$		$\frac{4}{9} \frac{\hat{s}^2 + \hat{u}^2}{\hat{t}^2}$
$q\bar{q} \rightarrow q'\bar{q}'$		$\frac{4}{9} \frac{\hat{t}^2 + \hat{u}^2}{\hat{s}^2}$
$q\bar{q} \rightarrow gg$		$\frac{32}{27} \frac{\hat{t}^2 + \hat{u}^2}{\hat{t}\hat{u}} - \frac{8}{3} \frac{\hat{t}^2 + \hat{u}^2}{\hat{s}^2}$

High Energy Limit: $|\hat{t}|$ fixed, $\hat{s} \rightarrow \infty$

t -channel dominance

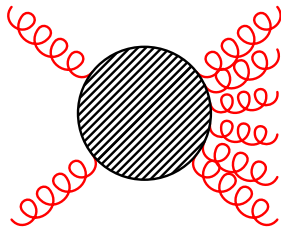
Example: $W+n$ -jet production at the LHC



$$\Delta y = y_{j_2} - y_{j_1}, y_W, y_{j_2} \geq 1, y_{j_1} \leq -1$$

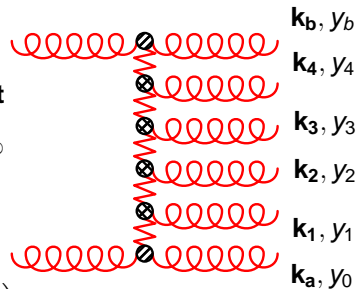
The Possibility for Prediction of n -jet Rates

The Power of Reggeisation



High Energy Limit

$|\hat{t}| \text{ fixed, } \hat{s} \rightarrow \infty$



$$\mathcal{A}_{2 \rightarrow 2+n}^R = \frac{\Gamma_{A'A}}{q_0^2} \left(\prod_{i=1}^n e^{\omega(q_i)(y_{i-1}-y_i)} \frac{V^{J_i}(q_i, q_{i+1})}{q_i^2 q_{i+1}^2} \right) e^{\omega(q_{n+1})(y_n-y_{n+1})} \frac{\Gamma_{B'B}}{q_{n+1}^2}$$

$$q_i = \mathbf{k}_a + \sum_{j=1}^{i-1} \mathbf{k}_j$$

NLL: Fadin, Fiore, Kozlov, Reznichenko

Resum to all orders in the perturbative At LL only gluon production; at expansion terms of the form NLL also quark-anti-quark pairs

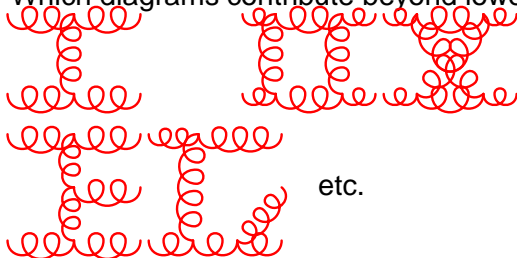
$$\left(\alpha_s \ln \frac{\hat{s}_{ij}}{|\hat{t}_j|} \right)$$

produced.
Prediction of **any-jet** rate possible.

FKL at Leading Logarithmic Accuracy

Fadin, Kuraev, Lipatov

Which diagrams contribute beyond lowest order?



All these contributions can be calculated using **effective vertices** and propagators for the **reggeized gluon**.

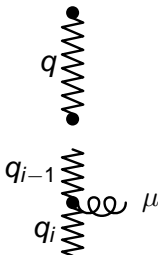


General form proved using s-channel unitarity and a set of bootstrap relations [NLL: Fadin, Fiore, Kozlov, Reznichenko](#)

FKL formalism

(Fadin, Kuraev, Lipatov)

FKL: Identification of the **dominant contributions** to the **perturbative series** for processes with two large (perturbative) and disparate energy scales $\hat{s} \gg |\hat{t}|$ (\hat{s} : E_{cm}^2 , \hat{t} : p_{\perp}^2)



$$\frac{1}{q^2} \exp(\hat{\alpha}(q)\Delta y)$$

$$C_L^\mu(q_{i-1}, q_i)$$

Framework valid within the Multi Regge Kinematic (MRK) of

$$y_0 \gg y_1 \gg \dots \gg y_2, \quad |k_{i\perp}| \approx |k_{j\perp}|, \quad q_i^2 \approx q_j^2$$

Interesting fact: Reproduces the **MHV Parke-Taylor amplitudes** in the **High Energy Limit**

Calculating effective vertices

The Ingredients of the NLL Vertex

$$V(\mathbf{q}_1, \mathbf{q}_2) = \left| \text{diagram 1} \right|^2 + \int d\mathcal{P} \left| \text{diagram 2} \right|^2 + \int d\mathcal{P} \left| \text{diagram 3} \right|^2$$

The diagrams represent different types of effective vertices:

- Diagram 1: A circle with a cross-hatch pattern, connected to four wavy lines (two incoming, two outgoing).
- Diagram 2: A circle with diagonal hatching, connected to four wavy lines (two incoming, two outgoing).
- Diagram 3: A circle with diagonal hatching, connected to four wavy lines (two incoming, two outgoing).

Two methods for obtaining the vertices at NLL:

- Fadin & Lipatov:

$$\text{diagram 3} = \text{diagram 4} + \text{diagram 5}$$

The diagrams represent different types of effective vertices:

- Diagram 4: A circle with diagonal hatching, connected to four wavy lines (two incoming, two outgoing).
- Diagram 5: A circle with diagonal hatching, connected to four wavy lines (two incoming, two outgoing).

- V. Del Duca:

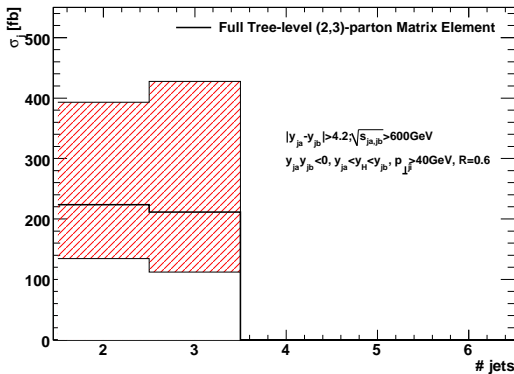
$$\text{diagram 3} = \lim \left(\text{diagram 6} / \left(\text{diagram 7} \times \text{diagram 8} \right) \right)$$

The diagrams represent different types of effective vertices:

- Diagram 6: A circle with diagonal hatching, connected to four wavy lines (two incoming, two outgoing).
- Diagram 7: A circle with diagonal hatching, connected to four wavy lines (two incoming, two outgoing).
- Diagram 8: A circle with diagonal hatching, connected to four wavy lines (two incoming, two outgoing).

Case study and Validation

Tree level results for $pp \rightarrow \text{Higgs} + \text{jets}$



Necessary to understand multi-emission topologies in order to

- cleanly extract WBF signal (c. jet veto, angular dist. of jets, ...)
- use H+jets as a discovery channel

using WBF cuts: $\sigma_{hjj} = 223^{+170}_{-89} \text{ fb}$, $\sigma_{hjjj} = 211^{+217}_{-99} \text{ fb}$.

Higgs Boson plus $n \geq 2$ jets in the HE limit



Extract the effective Higgs Boson vertex using the method of VDD

Only two diagrams contribute to the process Higgs Boson plus 3 jets in the High Energy Limit!

Some contributions have vanishing HE limit. . .

$pp \rightarrow h + \text{jets}$ with vanishing HE limit

sub-processes not contributing at all:

$u\bar{u} \rightarrow ghg(g), gg \rightarrow uh\bar{u}(g)$

or not in special rapidity configurations (at LL):

$gu \rightarrow uhg, ud \rightarrow dhu, gu \rightarrow ghug, \dots$

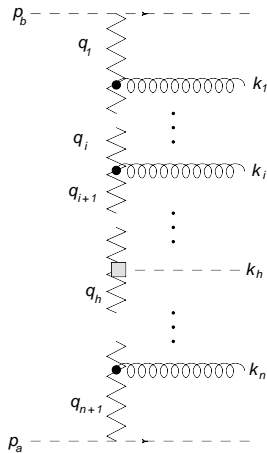
Total contribution from full ME of these contributions:

$$\sigma_{hj}^{\text{van.HE.limit}} = 0.5fb$$

$$\sigma_{hjj}^{\text{van.HE.limit}} = 20fb$$

Contributes less than 10% of the cross section. The HE limit will approximate the remaining configurations (will later add back the missing pieces by matching to the fixed order results)

The Scattering Amplitude



The Scattering Amplitude

$$\begin{aligned}
 i\mathcal{M}_{\text{HE}}^{ab \rightarrow p_0 \dots p_j h p_{j+1} p_n} &= 2i\hat{s} \\
 &\cdot (ig_s f^{ad_0 c_1} g_{\mu_a \mu_0}) \\
 &\cdot \prod_{i=1}^j \left(\frac{1}{q_i^2} \exp[\hat{\alpha}(q_i^2)(y_{i-1} - y_i)] (ig_s f^{c_i d_i c_{i+1}}) C_{\mu_i}(q_i, q_{i+1}) \right) \\
 &\cdot \left(\frac{1}{q_h^2} \exp[\hat{\alpha}(q_h^2)(y_j - y_h)] C_H(q_{j+1}, q_h) \right) \\
 &\cdot \prod_{i=j+1}^n \left(\frac{1}{q_i^2} \exp[\hat{\alpha}(q_i^2)(y'_{i-1} - y'_i)] (ig_s f^{c_i d_i c_{i+1}}) C_{\mu_i}(q_i, q_{i+1}) \right) \\
 &\cdot \frac{1}{q_{n+1}^2} \exp[\hat{\alpha}(q_{n+1}^2)(y'_n - y_b)] (ig_s f^{bd_{n+1} c_{n+1}} g_{\mu_b \mu_{n+1}})
 \end{aligned}$$

The Traditional Implementation Using the BFKL Eqn*

Adding one emission \rightarrow emergence of extra factor in $|\mathcal{M}|^2$ of

$$\frac{-C^{\mu_i} \cdot C_{\mu_i}}{t_i t_{i+1}} \rightarrow \frac{4}{p_{i\perp}^2}$$

in the ultimate MRK limit. Taking into account contraction of colour factors, the addition of an emission leads to the following factor in the colour and spin summed and averaged square of the matrix element

$$\frac{4 g_s^2 C_A}{p_{i\perp}^2}$$

Only transverse degrees of freedom left!

*Now is a good time to take a nap - in a few minutes I will ask you to forget all about the BFKL eqn.

The Traditional Implementation Using the BFKL Eqn*

$$\left| \mathcal{M}^{gg \rightarrow hgg} \right|^2 = \frac{4\hat{s}^2}{N_c^2 - 1} \frac{C_A g_s^2}{p_{0\perp}^2} \left| C_{HEL}^H(-p_{0\perp}, p_{1,\perp}) \right|^2 \frac{C_A g_s^2}{p_{1\perp}^2}$$

$$\left| \mathcal{M}^{gg \rightarrow hggg} \right|^2 = \frac{4\hat{s}^2}{N_c^2 - 1} \frac{C_A g_s^2}{p_{0\perp}^2} \left| C_{HEL}^H(q_{a\perp}, q_{b,\perp}) \right|^2 \frac{4}{p_{1\perp}^2} \frac{C_A g_s^2}{p_{2\perp}^2}$$

⋮

⋮

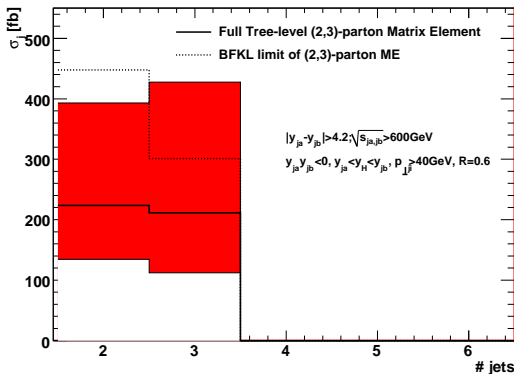
⋮

$$\frac{d\hat{\sigma}_{gg \rightarrow g \cdots h \cdots g}}{dp_{a\perp}^2 dy_a dp_{b\perp}^2 dy_b dp_{H\perp}^2 dy_H} = \int d^2 q_{a\perp} d^2 q_{b\perp} \left(\frac{\alpha_s N_c}{p_{a\perp}^2} \right) f(-p_{a\perp}, q_{a,\perp}, \Delta y_{aH})$$

$$\cdot \left| C_{HEL}^H(q_{a,\perp}, q_{b,\perp}) \right|^2 f(q_{b\perp}, p_{b,\perp}, \Delta y_{Hb}) \left(\frac{\alpha_s N_c}{p_{b\perp}^2} \right)$$

$$\omega f_\omega(\mathbf{k}_a, \mathbf{k}_b) = \delta^{(2+2\epsilon)}(\mathbf{k}_a - \mathbf{k}_b) + \int d^{2+2\epsilon} \mathbf{k} \mathcal{K}_\epsilon(\mathbf{k}_a, \mathbf{k} + \mathbf{k}_a) f_\omega(\mathbf{k} + \mathbf{k}_a, \mathbf{k}_b).$$

Comparison between BFKL and Full Matrix Element



V. Del Duca, C. White, JRA

Not convincing*. Can obviously match to FO, but better also improve resumⁿ!

* And this is even the energy and momentum conserving variant of BFKL - please ask about this point if you want to see something crazy. It is actually a very important point.

Improving the Framework*

FKL amplitudes:

$$i\mathcal{M}_{\text{HE}}^{ab \rightarrow p_0 \dots p_j h p_{j+1} p_n} = 2i\hat{s} \dots \prod_{i=1}^j \left(\frac{1}{q_i^2} \exp[\hat{\alpha}(q_i^2)(y_{i-1} - y_i)] \left(ig_s f^{c_i d_i c_{i+1}} \right) c_{\mu_i}(q_i, q_{i+1}) \right) \dots$$

Unmodified in MRK limit, but two supplementary guidelines for use outside the strict MRK limit:

- 1 Do not introduce new divergences
- 2 Do not apply the formalism where it fails

* Now would be a good time to wake up. Any time now. Please.

Improving the Framework*

FKL amplitudes:

$$i\mathcal{M}_{\text{HE}}^{ab \rightarrow p_0 \dots p_j h p_{j+1} p_n} = 2i\hat{s} \cdot \dots \prod_{i=1}^j \left(\frac{1}{q_i^2} \exp[\hat{\alpha}(q_i^2)(y_{i-1} - y_i)] \left(ig_s f^{c_i d_i c_{i+1}} \right) C_{\mu_i}(q_i, q_{i+1}) \right) \dots$$

Unmodified in MRK limit, but two supplementary guidelines for use outside the strict MRK limit:

- 1 Do not introduce new divergences
- 2 Do not apply the formalism where it fails

Using the full expression for the propagators in the formula above corresponds to *removing* some divergences from the full scattering amplitude (the collinear divergences), but not *moving* any divergences. This is different to the case where the MRK limit of invariants has been substituted (aka the BFKL eqn.), which displaces divergences within the phase space region of interest for the LHC (aka “diffusion problem”).

* Now would be a good time to wake up. Any time now. Please.

Improving the Framework*

FKL amplitudes:

$$i\mathcal{M}_{\text{HE}}^{ab \rightarrow p_0 \dots p_j h p_{j+1} p_n} = 2i\hat{s} \cdot \dots \prod_{i=1}^j \left(\frac{1}{q_i^2} \exp[\hat{\alpha}(q_i^2)(y_{i-1} - y_i)] \left(ig_s f^{c_i d_i c_{i+1}} \right) C_{\mu_i}(q_i, q_{i+1}) \right) \cdot \dots$$

Unmodified in MRK limit, but two supplementary guidelines for use outside the strict MRK limit:

- 1 Do not introduce new divergences
- 2 Do not apply the formalism where it fails

Using full expression for propagators automatically takes into account the dominant source of NLL corrections to *any* logarithmic accuracy. NLL corrections to Lipatov Vertex C^μ can restore the full propagator between two neighbouring gluons. We can restore the full propagator between all neighbouring gluons. Would need $N^n LL$ corrections to restore full propagators between $(n + 1)$ gluons.

*Now would be a good time to wake up. Any time now. Please.

Improving the Framework*

FKL amplitudes:

$$i\mathcal{M}_{\text{HE}}^{ab \rightarrow p_0 \dots p_j h p_{j+1} p_n} = 2i\hat{s} \cdot \dots \prod_{i=1}^j \left(\frac{1}{q_i^2} \exp[\hat{\alpha}(q_i^2)(y_{i-1} - y_i)] \left(ig_s f^{c_i d_i c_{i+1}} \right) C_{\mu_i}(q_i, q_{i+1}) \right) \cdot \dots$$

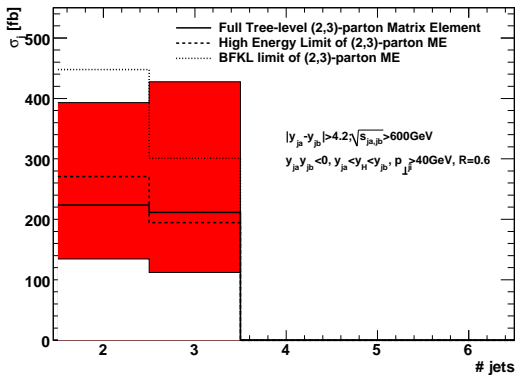
Unmodified in MRK limit, but two supplementary guidelines for use outside the strict MRK limit:

- 1 Do not introduce new divergences
- 2 Do not apply the formalism where it fails

Minimal interference: Insist just $-C^\mu C_\mu > 0$. Cuts out only a small region of phase space. Related to so-called *Kinematical Constraint* of CCFM eqn. (i.e. require dominance by transverse degrees of freedom)[actually allows for a check of the kinematic constraint directly on the formalism underpinning the BFKL eqn, instead of assuming the BFKL equation and then repairing with kin. cons.]

*Now would be a good time to wake up. Any time now. Please.

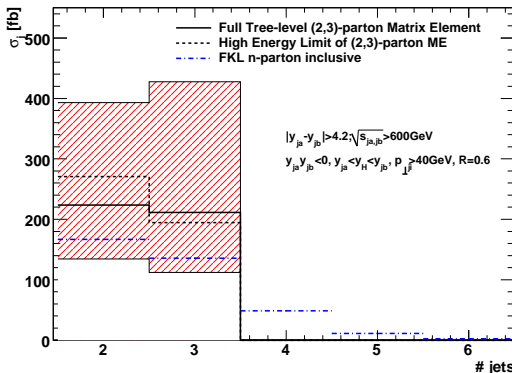
Comparison between FKL and Full Matrix Element



V. Del Duca, C. White, JRA

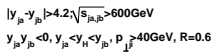
Difference between FKL (2 diagrams) and full result (10^3 diagrams) is much less than the renormalisation and factorisation scale uncertainty. Repair with matching corrections. We understand why the 3jet rate is better reproduced than the 2jet rate...

FKL All Order Resummation Incl. Matching



V. Del Duca, C. White, JRA

Can sum over n -parton inclusive samples (both real and virtual contributions included). Matching to the tree level n -parton matrix elements (mix R and $\ln R$ depending on whether or not the subprocess is vanishing in FKL descrip.)



V. Del Duca, C. White, JRA

Any central jet veto will obviously only operate on states with 3 and more jets. According to this calculation, it seems around 50% of total cross section would survive any additional jet veto (i.e. $p_T < 40\text{GeV}$).

Impact on Observables

$$A_\phi = \frac{\sigma(\phi_{j_a j_b} < \pi/4) - \sigma(\pi/4 < \phi_{j_a j_b} < 3\pi/4) + \sigma(\phi_{j_a j_b} > 3\pi/4)}{\sigma(\phi_{j_a j_b} < \pi/4) + \sigma(\pi/4 < \phi_{j_a j_b} < 3\pi/4) + \sigma(\phi_{j_a j_b} > 3\pi/4)}$$

Results from lowest order:

$A_\phi > 0$ (CP-even), $A_\phi \approx 0$ (CP-blind), $A_\phi < 0$ (CP-odd)

A_ϕ (2p/2j)	0.50
A_ϕ (3p/3j)	0.23
A_ϕ (FKL/ $\geq 2j$)	0.16
A_ϕ (FKL/ $\equiv 2j$)	0.27

Significant azimuthal decorrelation from higher orders real radiation - even when **not hard enough to be detected as jets!**

Outlook and Conclusions

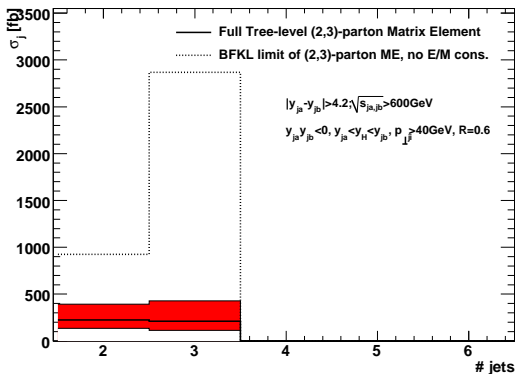
Conclusions

- Emerging framework for the study of processes with multiple hard jets
- Working implementation, including matching to the known fixed order results
- Impact many studies: jet correlations, missing (transverse) energy,...

Outlook

- H+jets studies being finalised; expect paper and code soon
- Implement other processes and test against Tevatron Data
- Les Houches Interface to study effects of showering
- Extend Studies to full NLL Accuracy
- ...

Thank you for asking that question. . .



Formulation valid for $\hat{s} \rightarrow \infty, |t|$ fixed. But $\hat{s} < s$ fixed at any collider! E/M conserv. not just “subleading corrections” in partonic scattering, but stops the evolution all together (even before the strict MRK limit is reached!).